**МИНОБРНАУКИ РОССИИ**

**САНКТ-ПЕТЕРБУРГСКИЙ ГОСУДАРСТВЕННЫЙ**

**ЭЛЕКТРОТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ**

**«ЛЭТИ» ИМ. В.И. УЛЬЯНОВА (ЛЕНИНА)**

**Кафедра АМ**

**ОТЧЕТ**

**по домашней работе №3**

**по дисциплине «Функциональный анализ»**

**Тема: Мера и интеграл.**

|  |  |  |
| --- | --- | --- |
| Студентка гр. 1384 |  | Пчелинцева К.Р. |
| Преподаватель |  | Коточигов А.М. |

Санкт-Петербург

2024

**ТЕОРЕТИЧЕСКИЕ ПОЛОЖЕНИЯ.**

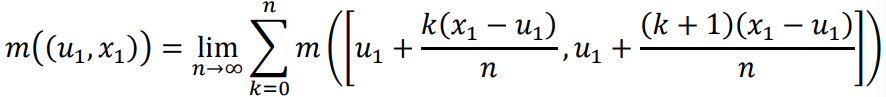
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Для любого

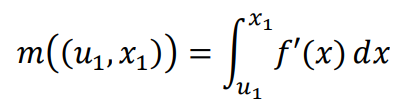
Для точки разрыва f:



Мера также может быть вычислена как интеграл:



Так как 𝑓(𝛽) − 𝑓(𝛼) = 𝑓′(𝛾)(𝛽 − 𝛼), 𝛼 < 𝛾 < 𝛽, предел является интегральной суммой, откуда



Мера аддитивна: 𝑚(𝐴 ∪ 𝐵) = 𝑚(𝐴) + 𝑚(𝐵), 𝐴 ∩ 𝐵 = ∅

**ВЫПОЛНЕНИЕ РАБОТЫ**

**Вариант 12.**

x0 = 0; x1 = ; x2 = ; x= 1

0 = 0; 1 = ; 2 = ; 3 = ; 4 = ; 5 = 2

*u0 = 0; u1 = ; u2 = ; u3 = ; u4 =* 1

*y0 = 2; y1= 1; y2 = 2; y3 = 4; y4 = 2*

**Задание 1**.

Вычислить 𝑘1, 𝑘2, 𝑘3, 𝑘4. Нарисовать график.

Из вида функции имеем: 𝑘1𝑥12 = 𝑣1. Отсюда 𝑘1 = 𝑣1/𝑥12 =

На интервале [𝑥1, 𝑥2) функция постоянна: 𝑘2 = 𝑣2

На интервале [𝑥2, 𝑥3] функция задаётся как 𝑘4 − 𝑘3 (1 − 𝑥)2, в крайних точках она равна 𝑣4 и 𝑣5. Отсюда находим 𝑘4 = 2, 𝑘3 = .

Функция 𝑓 задаётся как 𝑘1𝑥2 при 𝑥0 ≤ 𝑥 < 𝑥1. Как 𝑘2 при 𝑥1 ≤ 𝑥 < 𝑥2. Как 𝑘4 − 𝑘3 (1 − 𝑥)2 при 𝑥2 ≤ 𝑥 ≤ 𝑥3.

Итого имеем:

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Теперь построим график – рис. 1.

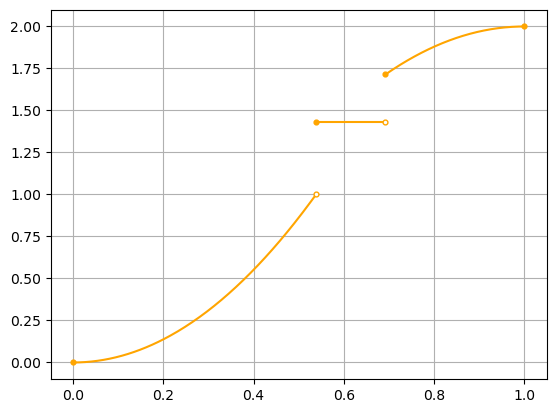


Рисунок 1 – График функции

Определим еще одну группу чисел:

|  |  |
| --- | --- |
|  | 0 |
|  |  |
|  |  |
|  |  |
|  | 1 |

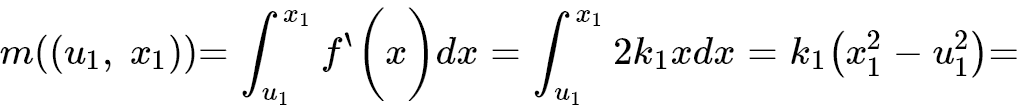
Выпишем значения функции во всех точках, задействованных в работе:

|  |  |
| --- | --- |
| ***x*** | ***f(x)*** |
| *x0 = 0* | 0 |
| *u1 = 2/9* | 7/44 |
| *x1 - 0* | 7/11 |
| *x1 + 0* | 12/11 |
| *u2 = 5/9* | 12/11 |
| *x2 - 0* | 12/11 |
| *x2 + 0* | 16/11 |
| *u3 = 5/6* | 41/22 |
| *x3 = 1* | 2 |

**Задание 2**.

Положим , . Показать, что .

Воспользуемся аддитивность меры и вычислим отдельно.

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for Microsoft Add-in"}

Можно заметить, что на любом интервале , где функция f(x) дифференцируема . Отсюда следует, что предел является интегральной суммой и равен интегралу (Римана):

можно вычислить аналогично, но надо иметь в виду, что производная равна 0.

{"mathml":"<math style=\"font-family:stix;font-size:16px;\" xmlns=\"http://www.w3.org/1998/Math/MathML\"><semantics><mstyle mathsize=\"16px\"><mi>f</mi><mfenced><msub><mi>x</mi><mn>1</mn></msub></mfenced><mo>-</mo><mi>f</mi><mfenced><mrow><msub><mi>x</mi><mn>1</mn></msub><mo>-</mo><mn>0</mn></mrow></mfenced><mo>=</mo><mfrac><mn>5</mn><mn>11</mn></mfrac></mstyle><annotation 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for Microsoft Add-in"}

Тогда

Проводя аналогичные вычисления, получим

Теперь вычислим

Теперь вычислим

Получили

**Задание 3.**

Вычислить , где непрерывная на [0; 1] функция, линейная на отрезках и заданная в точках излома

Построим график двух функций – рис. 2.

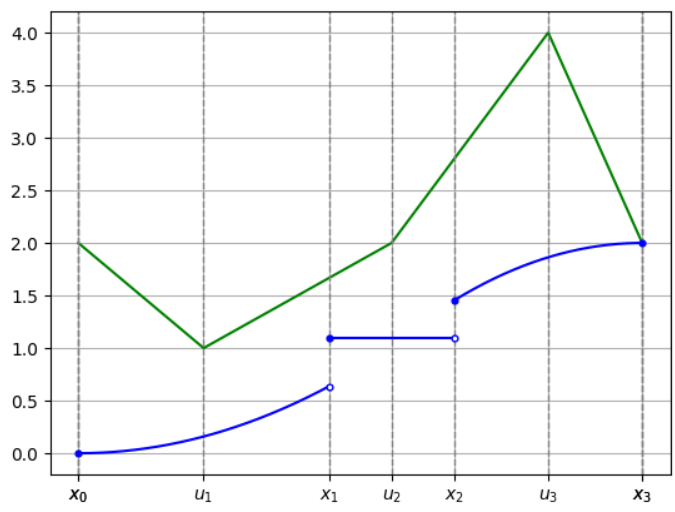
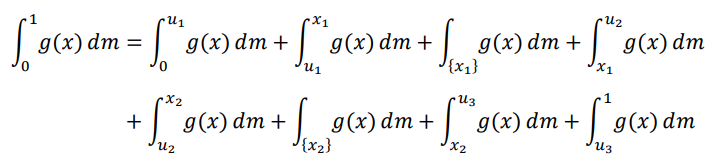
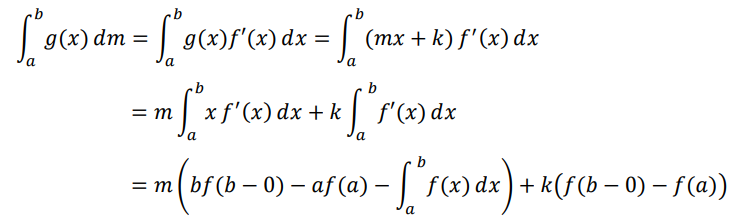


Рисунок 2 – График двух функций: g(x) - зеленый, f(x) - синий

Искомый интеграл удобно разбить по участкам, для которых обе функции g(x) и f(x) дифференцируемы.



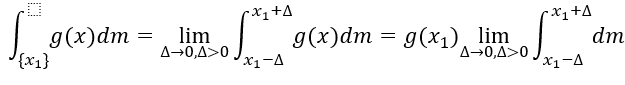
Интегралы на интервалах, где g(x) и f(x) дифференцируемы, можно записать так:



Найдем для всех участков линейной функции g(x) коэффициенты m и k:

|  |  |  |
| --- | --- | --- |
| = |  |  |
|  |  |  |

Интеграл в точке вычисляется через предел:



Получим формулу:

Вычислим все интегралы:

Просуммируем интегралы:

**Задание 4**.

Вычислить норму g(x) в пространстве , .

Сначала вычислим , точки разбиения интеграла будут такими же, как в прошлом пункте.

Посчитаем слагаемые:

Просуммируем интегралы

**Задание 5.**

Существует ли линейная функция , ортогональная функции g(x) в пространстве .

Посчитаем интеграл в первом слагаемом, аналогично предыдущим пунктам разделив отрезок на те же участки:

Найдем слагаемые:

Суммируем:

Имеем

Так, линейная функция ортогональна функции 𝑔(𝑥) в при любом b